Pole Tracking Method by Adaptive Hereditary Computation

Gibran Etcheverry and Carlos A. Reyes-García

INAOE, Department of Computer Science,
Enrique Erro 1, 72840, Tonantzintla, Mexico

Abstract. In the study of nonstationary signals, visualization of the time-frequency representation is a key figure for understanding the signal frequency content and its point of emergence within time. On the one hand, we can find the well studied non parametric techniques based on the Wigner-Ville distribution, employed on the telecommunications and radar signal analysis. On the other hand, we find the parametric modeling of the frequency content of a nonstationary behavior by means of an autoregressive (AR) filter, which throught its polynomial coefficients, leads us to the filter poles and the spectral components they represent. The adaptive hereditary computation technique, based on the acquisition of the time variable correlation function of the signal through an autoregressive moving average (ARMA) model, allows us a fine pole tracking from signal contiguous sections. Two validation examples are shown: a bat sonar signal for comparison with Wigner-Ville approach, and a normal newborn cry signal for the tracking ability of highly nonstationary signals.

1 Introduction

Nonstationary signals are those which present a time variable change of their statistical properties and frequency content [1]. For instance, we can find them in the areas of geology on seismic events [2], of telecommunications on frequency modulation, of radar on echo signal frequency shift [3], and of biomedicine over Heart Rate Variability (HVR) studies [4], electroencephalogram (EEG) characterization [5], newborn cry [6] and children speech analysis [7], among others.
The wide spread presence of these kind of signals has required to study them deeply in order to understand their behavior; for example, some of them present a chaotic sequence of frequency content in time like biomedical signals, some others a linear or nonlinear frequency shift while being transmitted or received in fm modulation or radar signals, respectively.
In this work, a pole tracking method allows us to follow a hyperbolic (nonlinear) frequency pattern obtained from a bat sonar signal and the nonstationnary behavior of a newborn cry record. This technique is a good choice in order to study narrowband and wideband signals [8]. The hereditary computation of the signal correlation function, gives us the possibility to determine the time variyng parameters between windows of time authomatically without windowing nor

overlapping. This adaptive task is carried out by an autorregressive moving average (ARMA) model, which employs the projected error from the estimated signal as the model input.

2 Adaptive Hereditary Computation

Adaptive hereditary computation has its origin in the area of systems identification by employing an ARMA model, in order to fit a stochastic realization to some measured output $y_\tau, \tau = 1, ..., t$ of an unknown system. The one-step ahead predictor therefore implemented, estimates the model coefficients without using a non linear optimization technique, as it is the case for gradient based or Gauss-Newton techniques [9]. Instead of it, recomputation of $\tau = 1, \ldots, T$ past samples is developed by using the parameters obtained at time t ; hence, the correlation function is time variyng and up to date, avoiding in this way calculation errors due to *incomplete* correlation terms obtained with standard estimation methods.

2.1 ARMA form

The one-step ahead predictor in ARMA form is written as follows:

$$\hat{y}_t = \sum_{i=1}^{n} a_i \hat{y}_{t-i} + \sum_{j=1}^{n} b_i \tilde{y}_{t-j} \tag{1}$$

where $\tilde{y}_t = y_t - \hat{y}_t$ and a_i,b_i are the autorregresive and input coefficients, respectively. This expression has beed derived from the simpler moving average (MA) form:

$$\hat{y}_t = \sum_{i=1}^{n} a_i y_{t-i} \tag{2}$$

which is an interesting point of view, considering the rational form (1), as a more efficient and rugged manner of modeling linear time series.

2.2 Hereditary computation

As it was stated in the begining of this section, the transient optimization approach can be developed to the price of hereditary computation of the model coefficients, presenting a linear t-growing memory of size nt, with n as the system dimension or delay. Hence, the ARMA form having these characteristics is written as:

$$\hat{y}^t_\tau = \sum_{i=1}^{n} a^t_i \hat{y}^{t-i}_{\tau-i} + b^t_i \tilde{y}^{t-i}_{\tau-i}, \quad \forall \tau = 1, \ldots, t. \tag{3}$$

In order to obtain the model parameters, it is necessary to employ an evaluation criteria of how well (3) performs; this criteria consists on minimizing the mean square error (MSE) between the time series y_t and the predictor $\hat{y}^t_\tau$:

$$J^t_T = E^t_T[(y_\tau - \hat{y}^t_\tau)^2] = \frac{1}{T} \sum_{\tau=t-T+1}^{t} (y_\tau - \hat{y}^t_\tau)^2 \quad \tau = 1, \ldots, T. \tag{4}$$

where T is the time horizon of *hereditary computation* or *re-computation* of the estimated samples. This means that the estimator *adapts* its horizon every T samples to get the time-variyng parameters a^t_i, b^t_i that characterize the signal of interest.
Derivating (4) with respect to the model parameters and separating terms to each side of the equality, leads us to the *normal equations*, see [10], which contain the time-varying correlation and intercorrelation terms:

$$\begin{bmatrix} \sum_{\tau=t-T+1}^{t} \hat{y}^{t-i}_{\tau-i}\hat{y}^{t-i}_{\tau-i} & \sum_{\tau=t-T+1}^{t} \hat{y}^{t-i}_{\tau-i}\tilde{y}^{t-i}_{\tau-i} \\ \sum_{\tau=t-T+1}^{t} \tilde{y}^{t-i}_{\tau-i}\hat{y}^{t-i}_{\tau-i} & \sum_{\tau=t-T+1}^{t} \tilde{y}^{t-i}_{\tau-i}\tilde{y}^{t-i}_{\tau-i} \end{bmatrix} \begin{bmatrix} a^t_i \\ b^t_i \end{bmatrix} = \begin{bmatrix} \sum_{\tau=t-T+1}^{t} y_\tau \hat{y}^{t-i}_{\tau-i} \\ \sum_{\tau=t-T+1}^{t} y_\tau \tilde{y}^{t-i}_{\tau-i} \end{bmatrix}$$

i=1. . . n.(5)

Once the aforementioned polynomal parameters a^t_i are calculated, it is possible to extract the system poles in the view of analyzing the frequency domain characteristics of the signal or system explored, as explained in next section.

3 Pole Tracking Method

Pole Tracking algorithms have found their path on the autorregresive (AR) models implemented for recursive system identification [11]. They are useful because they are simple to implement and have good spectral definition, mostly on wideband signals [8].
As it can be observed on expression (2), when replacing $y_t = \hat{y}_t + \tilde{y}_t$ to have (1), we realize that the numerator and denominator coefficients are the same [12]. In the case of analyzing a signal with changing mean value, the moving average coefficients will present an average of the autorregresive ones. Therefore, assuming this is not the case for simplicity, the moving average polynomial

equals to one. Then, the resulting transfer function for denominator polynomial coefficients (AR coefficients) computation is written as:

$$H(z) = \frac{1}{\sum_{i=1}^{n} a_i z^{-i}} \tag{6}$$

which can be factored to obtain the poles of the signal/system of interest:

$$H(z) = \frac{z^n}{\prod_{i=1}^{n}(z - z_i)} = \frac{z^n}{\prod_{i=1}^{n} \overrightarrow{P_i}} \tag{7}$$

where z_i are the poles of dimension $i = 1, \ldots, n$ and $\overrightarrow{P_i}$ the corresponding vectors from any point z in the complex plane to each of the n poles of (7), see [13].

The power spectral density (PSD) function can therefore be calculated throught the formula:

$$P(z) = \frac{\sigma^2}{(\prod_{i=1}^{n} |(z - z_i)|)^2 F_s} \tag{8}$$

where σ^2 is the signal variance, F_s the sampling frequency, $z = e^{j2\pi/F_s}$ and $z_i = e^{j2\pi f_i/F_s}$, para $i = 1, \ldots, n$.

The *pole tracking* task can be performed, with expression (7), by obtaining the frequency asociated to every pole of the model process; this is done by extracting the phase angle θ_i of the z_i, $i = 1, \ldots, n$ poles through the expression:

$$f_i = \frac{\theta_i}{2\pi} F_s \tag{9}$$

In order to study the spectral characteristics of (8), it is important to mention that the closer the pole is to the unit circle given by $z = e^{j2\pi/F_s}$, the higher its peak or contribution will be to the total PSD of the signal or system.

3.1 Model Dimension

Model dimension selection is an important matter in terms of good quality estimation of signal or system characteristics, due to the fact that a low model dimension could lead to poor results; while a high dimension selection might introduce undesired frequency components [8]. The work of Varghese et al. [14] reports probabilistic simulations for this purpose and for an specific case, which is an interesting approach with a certain complexity degree.
One possible trade off between complexity and signal analysis experience, is to employ first the projection matrix:

$$P_y = Y(Y^T Y)^{-1} Y^T \tag{10}$$

where Y is hankel and of size TxT, as it is commonly developed in filter estimation theory [15]:

$$\hat{Y} = P_y Y \tag{11}$$

where $\hat{Y}$ is a matrix of size TxT formed with shifted samples vectors.
Afterwards, the singular value decomposition (SVD) of Y yields the *system modes*; the amount of modes presenting a normalized value close to unity give the model dimension [16]. The resulting dimension through these operations depends on the size of the T-time window and the frequency content behaviour (which is in part infered by experience) of the signal, as it is explained on the results section.

3.2 Algorithm

The elements presented for analzing non stationary signals in this work are sumarized as follows:

1. Estimate and select the model order based on the SVD of (11).
2. Compute the model coefficients from normal equations (5).
3. Extract the poles from (6) in the view of (7).
4. Calcule the PSD in (8) and the frequencies associated to every pole in (9).

4 Results

4.1 Bat signal

The pole tracking method by hereditary computation has been tested over a bat sonar signal obtained from the Time-Frequency Toolbox [17] developed by the CNRS (Centre National de la Recherche Scientifique) and the Rice University. In addition, the toolbox provides a function `tfrpwv` that implements the pseudo-Wigner-Ville distribution, which is a discrete-time windowed frequency version of the Wigner-Ville distribution [3]. This function has been used on the bat signal in order to analyze, compare and validate the pole tracking method presented in this work for narrowband signals.
The employed bat sonar signal has been sampled at 230.4KHz and presents a range of frequencies from approximately 38-50KHz, considering values above 20 percent with respect to its maximum within the normalized power spectral density (PSD) plot on Fig. 1. This fact has been verified by using the matlab Fast Fourier Transform `fft` function to observe the signal frequency content without its progression in time.

The time-frequency analysis of the bat sonar signal, performed by using the pseudo-Wigner-Ville distribution function `tfrpwv`, yields the plot of Fig. 2. It is possible to observe in this figure that the bat sonar signal presents an hyperbolic frequencial shift within time from 57.6KHz to about 39KHz.

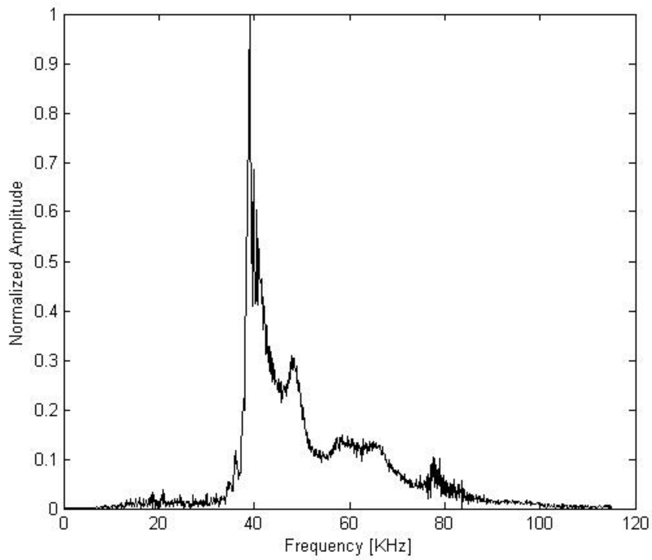

Fig. 1. PSD of the bat sonar signal.

The pole tracking method developed for time-frequency analysis along this document shows its performance on Fig. 3 with an hyperbolic frequency shift from about 55KHz to 38KHz. The model dimension employed for this analysis, as explained on subsection 3.1, was of $n = 2$ and it was obtained from the mean number of singular respresentative values, as well as from the performance comparison with the Wigner-Ville distribution method. It is possible to say, based on this information, that the pole tracking method accomplishes a fine preformance on depicting the time-frequency behavior of the bat sonar signal, and even goes a little further on the lower limit of the bat signal frequency content. This statement is supported when mirrowing the frequency content observed on Fig. 1 and the time-frequency behavior showed on Fig. 2 with Fig. 3.

4.2 Newborn Cry signal

In the view of valildating the hereditary computation of the AR coefficients with respect to the well known Linear Predictive Coefficients (LPC) method for nonstationary signal analysis [18], a non perturbed signal containing two sinusoids of 250Hz and 300Hz was simulated and the frequency estimated from the beforehand mentioned pole tracking method by employing each technique. The frequencies were obtained every 100ms from a one second signal sampled at a rate of 1000Hz. As it is observed on Figure 4, there is not error on the estimated

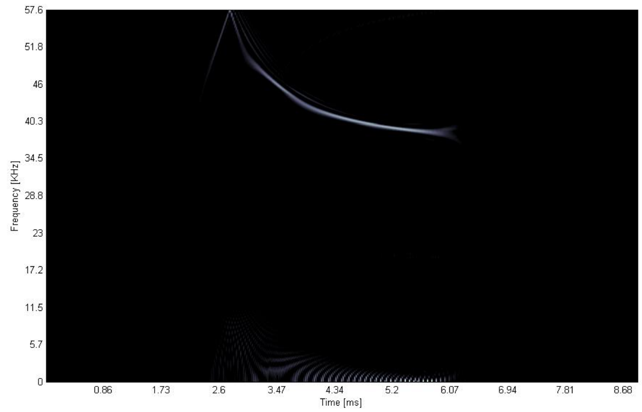

Fig. 2. Time-Frequency plot from Wigner-Ville distribution.

frequencies from the hereditary technique, whereas there is about a general 2 percent error from the LPC method. The `lcp` command from the `Matlab 7.6.0` software was used.
It has been mentioned that the strength and weakness of the LPC method are the ability to obtain stable models for speech synhtesis and its vocal tract description lack, respectively. On the other hand, the ARMA model is able to obtain a better result based on certain estimation techniques, but some of the resulting poles can lie outside the unit circle [18]. The ARMA model is employed this time combined to the hereditary approach for completing the autocorrelation function at every step [9] of a normal newborn cry signal, which is nonstationary because a considerable part of the power spectra is in the low frequencies [19] (pitch of 400-600Hz).
The normal newborn cry recording was sampled at $F_s = 8000Hz$ and treated in segments of time horizon $T = 20ms$. It is important to mention that there is no overlapping between segments nor windowing to carry out the analysis. The order or dimension used for the linear estimator was of $n = 10$, considering the experience on speech and newborn cry analysis [6], and observing the average of the obtained dimensions as explained on subsection 3.1. This allows to track the pitch and 4 formants due to the 5 pairs of conjugated poles found when none real pole appears.

On Fig. 5 we can observe the frequency components evolution in time of a normal newborn cry. When the pitch and 4 formants are present, a '+' is used; when some of the five components dissapear, a 'x' is drawn. It is possible to notice the characteristic random presence of fading periods [6].

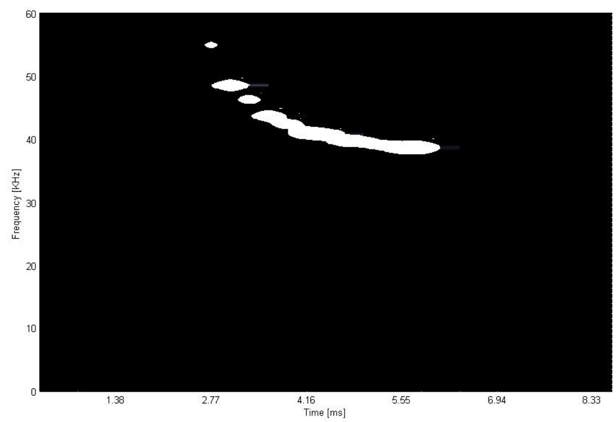

Fig. 3. Time-Frequency plot from Pole Tracking Method.

5 Conclusions and Future Work

In this work a new method for time-frequency analysis of narrowband and wideband nonstationary signals has been developed. It is based on the hereditary renewal and computation of past *T-samples*, once the correlation and intercorrelation products have been calculated, in the view of exctracting more accurate model coefficients which better describe the time series behavior. It is important to mention that this method allows an authomatic frequency information extraction throught the model poles, without needing signal *windowing* nor *overlapping* to capture the transient signal behavior, as it is frequently required [3]. Due to the linear approximation by sections that this method implements, it was possible to follow the nonlinear (hyperbolic) frequency shift behavior of a bat sonar signal. Besides it, a good time-frequency description of a quite nonstationary newborn cry signal was carried out. It is thougth that implementation of the hereditary computation combined to the Volterra series model, as it has been carried out for non linear systems identification [20], for biomedical [21] and speech [22] signals transient behavior analysis, will lead to usefull results due to the possibility of modeling nonlinear behaviors (bursts for instance, see [19]) contained on the linear estimation error by only using an ARMA or an LPC method.

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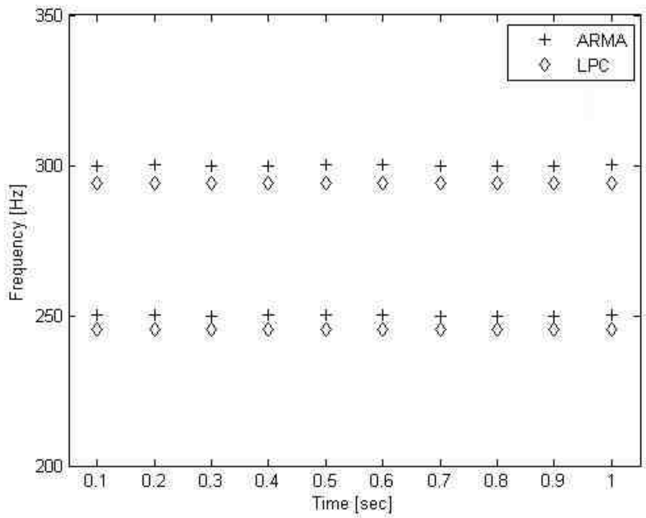

Fig. 4. Spectrogram of two sinusoids.

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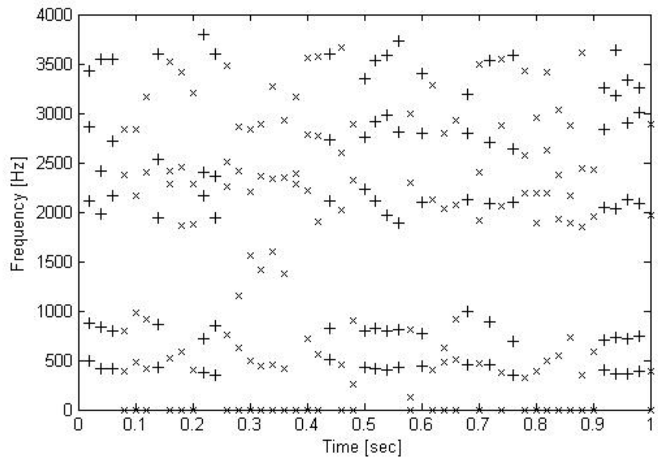

Fig. 5. Normal Newborn Cry.

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